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Allocating Assets in Climates of Extreme Risk

A New Paradigm for Stress Testing Portfolios

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Introduction

After predatory financial markets dismembered Lehman Brothers in the autumn of 2008, investors took refuge in US Treasuries. The rich returns to US sovereign bonds, however, were not evenly distributed across the market. Prices of Treasury Inflation Protected Securities (TIPS), for which the principal and interest payments are indexed to the CPI, fell dramatically as their nominal counterparts rose in value. Near-term "breakeven" or "expected" inflation, which is the difference between nominal and real yields, plummeted to –6.5%.[1]

The autumn 2008 divergence between TIPS and nominal Treasuries was unusual: the correlation between these asset classes has historically hovered around 0.9, but fell to a low of 0.4 in November 2008.[2] The divergence was also impactful, since TIPS account for roughly 10% of the US sovereign bond market, representing hundreds of billions of dollars in outstanding principal. Even after the fact, backward-looking statistical risk models are unfit to assess the likelihood of a dramatic shock to breakeven inflation. It is nevertheless possible to account for a shock of this type in an investment process.

Investors stress their portfolios to analyze the impact of extreme events, which tend to lie outside the purview of statistical risk measures. Stress tests can detect a portfolio's vulnerabilities and assess its expected reaction to market scenarios, and consequently can add significant value to an investment process. However, it can be challenging to determine and implement the most salient scenarios. Furthermore, the output of many stress tests is expressed in terms of profit and loss (P&L), and this information is not directly actionable. The investor must translate P&L into modified portfolio weights.

In this article, we address both of these issues. First, we introduce a structured set of tools that enable investors to envision and administer extreme historical and hypothetical scenarios. We show how to take account of historical and hypothetical covariance matrices in scenario construction, and we provide examples that demonstrate the substantial impact of doing so. In short, the risk climate can and should be incorporated in a stress test.

Second, we provide a means to incorporate the output of a portfolio stress test directly into an investment decision, which ultimately boils down to a tradeoff between the competing objectives of minimizing risk and maximizing return. In the examples provided below, we achieve this with a scenario-constrained mean-variance optimization. However, as discussed in Cases 4 and 5, we can incorporate extreme risk and other non-Gaussian effects in a stress test. Below, we review the standard framework for stress testing a portfolio. Subsequently, we sketch our new paradigm and provide numerous examples that illustrate its features.

[1] Inflation rates in the US have been overwhelmingly positive over the past century. The lowest inflation rates, roughly –5%, occurred in the mid-1920s and again in the mid-1930s. Since 1950, the rate of inflation has not fallen below 1%. The low level of breakeven inflation in autumn 2008 may be attributed to market-wide fears of a 1929-style deflation.

[2] A time series of correlation between TIPS and Nominal Treasuries is shown in Exhibit A1 of Appendix A.

Standard Stress Testing Methodology

Standard stress tests apply a market shock to an existing portfolio, and they analyze the impact in terms of the overall portfolio P&L. In the simplest case, a manager may wish to see how much his portfolio would lose if the term structure of interest rates were to rise by 2% across all maturities. All instruments with direct exposure to interest rates are revalued at the shocked levels, and the resulting portfolio profit or loss associated with this rate hike is computed.

However, dramatic rate hikes do not happen in isolation. This leads to a more comprehensive way to analyze the impact of rising interest rates on a portfolio: a factor or asset covariance matrix is used to apply a narrowly defined shock to assets that are not explicitly affected. While equities, for example, are not explicitly affected by yield curve movements, there is an expected impact that is implied by the correlations between equities and bonds. For completeness, we review the standard framework for estimating the expected impact.

Consider a collection X of returns to assets or asset classes that an investor wishes to stress. Let Y be a second (non-overlapping) collection of returns to assets, asset classes, or risk factors that can be shocked directly. Suppose that together, X and Y follow a joint normal distribution with covariance matrix Σ. In this setup, the expected impact on X of a shock to Y can be expressed as the mean of a conditional multivariate normal distribution.

$$E(X \mid Y) = E(X) + \Sigma_{XY} \Sigma_{YY}^{-1} (Y - E(Y)) \tag{1}$$

The unconditional means $E[X]$ and $E[Y]$ depend on the particular assets and factors in question, the time horizon for the shock, and the details of the ambient economic regime. They can be specified as part of the stress test, and for simplicity, we set them to zero in the examples below. As such, the expected shock to X, conditional on a shock to Y, depends only on the magnitude of the direct shock and the covariance matrix Σ.

Suppose the US Equity Market, Y, loses 10%. Using Formula (1), the expected P&L of X, say nominal US Treasuries, is the covariance of US Equities and nominal US Treasuries (Σ_{XY}) divided by the variance of the US Equity Market (Σ_{YY}), multiplied by the –10% shock magnitude. In other words, the implied P&L of nominal US Treasuries is the beta of nominal US Treasuries to Equities multiplied by –0.1.[3] Using an EWMA covariance matrix with a 21-day half-life estimated on September 9, 2009, the annualized daily covariance of US Equities with nominal US Treasuries is $-49.0\%^2$, while the annualized daily variance of US Equities is $391.3\%^2$. The P&L implied for nominal US Treasuries is thus $(-49.0/391.3) * (-0.10) = 1.25\%$.

[3] The simplicity of the calculation stems from the fact that we directly shocked a single asset class. In the case of a multivariate shock, the calculation of the expected P&L is more complicated. In essence, the set X of asset classes that we want to stress is projected onto the set of shocked asset classes Y, and the coefficients from this multivariate regression are used in tandem with the shock magnitude to reflect the forecast profit or loss to each asset class in X. Note that the forecast P&L depends only on the covariance matrix and the shocks, and it is independent of portfolio weights. This is an essential property for our inclusion of forecast P&L estimates into the asset allocation problem.

Expanding the Concept of a Stress Test

Following standard practice, we apply a shock to specific asset classes, and we infer the impact on other asset classes using Formula (1). Note however, that Formula (1) relies on a covariance matrix. So, in addition to explicit asset class shocks, we shock correlations between asset classes and asset class volatility levels as part of stress scenario generation. Through a series of examples, we analyze the impact of changes to a covariance matrix on P&L, and we show how to modify weights to optimally mitigate the effect of the shock. In the last two examples, we broaden the concept of risk to include non-Gaussian effects that are not fully explained by a covariance matrix, and we demonstrate a substantial impact on P&L. Throughout this paper, we define a scenario as the combination of a risk forecast with a set of explicit asset class shocks.

Our examples are based on the problem of allocating assets in a stressful situation. We develop two schematic scenarios. The first is an extreme deflationary scenario that is motivated by the autumn 2008 divergence of Nominal Treasuries and TIPS.[4] Specifically, we posit a 5% profit to Nominal Treasuries accompanied by a 5% loss to TIPS. The second is an extreme inflationary scenario, in which the value of a nominal investment decays, but an inflation adjusted investment is protected. Our inflationary shock is represented by a 5% profit to TIPS and a 5% loss to Nominal Treasuries.

Modifying an Asset Allocation to Account for Stress

Consider a plan for which the current allocation is given below. We use market indices to represent the six asset classes in the allocation; they are listed in Exhibit A3 of Appendix B. Our stress testing framework enables investors to both analyze the impact of scenarios on the plan and to modify the weights of the asset classes to take account of the impact.

Exhibit 1: Initial Plan Allocation. This allocation will be used as the starting point for all examples unless otherwise stated.

	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Weight	**50%**	**10%**	**10%**	**10%**	**10%**	**10%**

The framework for using forecast scenarios to perturb an existing asset allocation decision is a 3-step process.

- **Specify the Scenario**: Establish a forecast covariance matrix to represent the risk climate.(In our Gaussian examples, the risk climate is described fully by a covariance matrix, while in our more general empirical framework a covariance matrix is used in conjunction with additional information.) Explicitly shock a subset of the asset classes, and using the

[4]A recent history of *breakeven inflation* is described in Appendix D and shown in Exhibit A12-A14.

conditional multivariate framework described above in (1), infer profits and losses for the remaining asset classes.

- **Reverse Optimize:** Determine the expected asset class returns (alphas) that are consistent with the initial asset allocation, forecast covariance matrix, and level of risk aversion. These are the alphas for which the initial allocation is mean-variance optimal, given the investor's level of risk aversion and forecast covariance matrix.[5]
- **Perturb the Initial Allocation**: Mean-variance optimize the allocation subject to the following constraint: total portfolio loss under the specified scenario must be less than or equal to a specified bound.

Because the expected P&L for a given asset class is independent of the weight of that asset class, and since the overall profit or loss to the aggregate portfolio is a weighted function of the profits and losses of the individual asset classes, total portfolio profit or loss can be expressed as a linear constraint. The asset allocation problem can be succinctly expressed as a quadratic program.

Max $\alpha' w - \lambda w' V w$ such that

$\sum_i w_i = 1$	Full Investment	(2)
$w_i \geq 0 \ \ \forall i$	Long Only Positions	
$w' S \geq$ Max Loss	Constraint on loss from scenario	

In Formula (2), w represents the vector of allocation weights for each asset class, V represents the asset class covariance matrix, α represents a vector of expected asset class returns implied by the initial allocation and risk climate, and S represents the forecast impact vector of an imposed scenario across all asset classes. We explicitly shock a subset of the asset classes represented by S, and we infer implicit shocks to the remaining asset classes.[6] Note that other constraints, including turnover or position level bounds, can be incorporated into this formulation. In the event that the constraint on maximum loss is non-binding, that is to say, if the forecast profit and loss associated with the initial allocation does not exceed the stated maximum level, then the quadratic program in Formula (2) returns the initial portfolio.

[5] The reverse optimized alphas serve as a peg to the initial asset allocation. As such, they are different in character from exogenously supplied alphas that directly incorporate investors' views. It is possible to modify the process to include exogenous alphas and to use turnover constraints (or other means) to stay close to the initial allocation. Another alternative, which we explore in Case 3, is to modify the initial allocation in response to the scenario being applied, so that the reverse optimized alphas now peg to an alternate allocation.

[6] In all the examples below, we directly shock a subset of the variables in S, and we infer shocks to the remaining variables in the context of a *risk climate*. In Cases 1-3, the risk climate is embodied in a covariance matrix, and unspecified shocks are determined by Formula (1). To be precise, variables shocked explicitly correspond to Y, and the implicit shocks are given by $X = \Sigma_{XY}\Sigma_{YY}^{-1}Y$. In Cases 4 and 5, a non-Gaussian, empirically based method is used to infer X from Y.

Incorporating a Risk Climate in a Stress Test

Extreme events occur in both high- and low-volatility regimes, and the impact of an extreme event on a portfolio depends on the regime. In the examples below, we include a regime, or risk climate in a scenario, and illustrate the impact of doing so. Our initial examples feature a Gaussian framework, where risk is wholly defined by a covariance matrix and there is an implicit assumption of normality. However, with the incorporation of additional historical information, our methods extend naturally beyond the Gaussian distribution. Our final two examples illustrate how non-Gaussian extensions might work.

Case 1: Allocating Assets in a Stable Risk Climate

Roughly 3100 US market daily observations dating from 1997 to 2009 were equally weighted to forecast our stable asset class covariance matrix on September 9, 2009.[7]

Exhibit 2: Stable and Crisis Covariance Matrices. This exhibit highlights the substantial difference in volatilities and correlations that can emerge during times of crisis. The top panel provides stable volatilities and correlations for six US asset classes as of September 9, 2009, the estimation is based on twelve years of equally weighted daily data, while the bottom panel provides crisis volatilities and correlations for the six US asset classes estimated using a responsive 21-day exponentially weighted moving average as of November 20, 2008.

STABLE	Volatility (annualized)	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Equities	**21.67%**	**1.00**					
REITs	**31.51%**	**0.63**	**1.00**				
Nominal Treasuries	**4.92%**	**–0.26**	**–0.18**	**1.00**			
High Yield Bonds	**4.98%**	**0.19**	**0.10**	**–0.01**	**1.00**		
TIPS	**5.59%**	**–0.18**	**–0.14**	**0.71**	**0.10**	**1.00**	
Commodities	**16.78%**	**0.00**	**0.02**	**-0.02**	**0.02**	**-0.01**	**1.00**

[7]Equally weighting approximately 12 years of daily observations is one naïve approach to estimating a stable risk forecast. An alternative would be to construct and equal-weight a sample of observations taken from selected periods of market tranquility.

CRISIS	Volatility (annualized)	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Equities	**67.23%**	**1.00**					
REITs	**109.47%**	**0.86**	**1.00**				
Nominal Treasuries	**8.46%**	**–0.44**	**–0.41**	**1.00**			
High Yield Bonds	**18.80%**	**0.40**	**0.22**	**–0.35**	**1.00**		
TIPS	**11.62%**	**–0.01**	**–0.11**	**0.42**	**0.41**	**1.00**	
Commodities	**31.82%**	**0.11**	**0.13**	**-0.30**	**0.09**	**-0.01**	**1.00**

Consider the deflationary and inflationary scenarios described above. In the deflationary scenario, TIPS fall by 5% and Nominal Treasuries increase by 5% and in the inflationary scenario, the directions of the shocks are reversed. Exhibit 3 shows these explicit shocks, as well as the Formula (1) implicit shocks to other asset classes under the stable covariance regime described above. Also in Exhibit 3 are the reverse optimization implied alphas---these are the asset class expected returns that make the plan's existing weights optimal given the stable covariance regime and a risk aversion parameter of 0.0075[8]. The reader may notice that the implicit shocks for the inflationary scenario are equal in magnitude but of opposite sign to those for the deflationary scenario. This is a direct function of the symmetry in our choice of explicit shocks to represent these extreme economic scenarios.[9]

[8] A value of 0.0075 is arbitrarily selected as a risk aversion coefficient for this analysis. Note that because the mean variance objective function in our framework is based on implied alphas, which themselves are a function of risk aversion and serve as a peg to the initial allocation, the value of the risk aversion coefficient does not impact optimal weights.

[9] Note we define implicit shocks as $X = \Sigma_{XY}\Sigma_{YY}^{-1}Y$. If we let Y_D and X_D denote the explicit and implicit shocks for our above deflationary scenario, and Y_I and X_I as the explicit and implicit shocks for our above inflationary scenario, we have the following: $Y_I = Y_D[-1]$, $X_D = \Sigma_{XY}\Sigma_{YY}^{-1}Y_D$ and $X_I = \Sigma_{XY}\Sigma_{YY}^{-1}Y_I$. By substitution, we have $X_I = \Sigma_{XY}\Sigma_{YY}^{-1}Y_D[-1] = X_D[-1]$.

Exhibit 3: Implied alphas and Explicit and Implicit shocks for six US asset classes. This table highlights the characteristics of stress scenarios in different economic regimes. The implied alphas are reverse optimized from the September 9, 2009 stable covariance matrix shown in Exhibit 2 and the initial asset allocation in Exhibit 1. We explicitly shock Nominal Treasuries and TIPS to reflect disinflationary and inflationary scenarios, and we use the September 9, 2009 stable covariance matrix to imply shocks to the remaining four asset classes.

	Allocation implied alpha	Deflationary		Inflationary	
		Explicit	Implicit	Explicit	Implicit
Equities	**0.0164%**		**–6.76%**		**6.76%**
REITs	**0.0183%**		**–3.66%**		**3.66%**
Nominal Treasuries	**–0.0007%**	**5.00%**	**5.00%**	**-5.00%**	**-5.00%**
High Yield Bonds	**0.0007%**		**–1.74%**		**1.74%**
TIPS	**–0.0005%**	**–5.00%**	**–5.00%**	**5.00%**	**5.00%**
Commodities	**0.0018%**		**-0.49%**		**0.49%**

Using the stable covariance matrix, implied alphas, and explicit and implicit shocks, the quadratic program outlined in Formula (2) can be solved with a no-loss constraint to obtain the perturbed allocations shown in Exhibit 4. It is no surprise that the weight of Nominal Treasuries increases in the deflationary scenario. Historically, Nominal Treasuries have been a safe haven during market disruptions. Note that there is no perturbation required to satisfy a zero-loss constraint from our inflationary scenario. In our stable risk climate, Nominal Treasuries are negatively correlated with Equities and REITs. Consequently, a negative shock to Nominal Treasuries implies a gain for Equities and REITS, and the initial allocation is not adversely impacted by our inflationary scenario. More generally, the P&L of the shock, and the perturbed allocation in Exhibit 4, depend on the stable risk climate, which is based on average co-behavior of the asset classes between 1997 and 2009.

Exhibit 4: Optimal weights for six US asset classes under the deflationary and inflationary scenarios in Exhibit 3. We solve the quadratic program outlined in Formula 2 under a constraint that the portfolio incurs no loss. This optimization is driven by the implied alphas in Exhibit 3 and the stable covariance as of September 9, 2009 in Exhibit 2. The deflationary scenario slants the allocation toward Nominal Treasuries. The zero-loss constraint is not binding for our inflationary scenario, so the initial allocation is optimal and no perturbation occurs.

	Initial allocation	Zero-loss constraint	
		Deflationary shock	Inflationary shock
Equities	**50%**	**33.29%**	**50.00%**
REITs	**10%**	**12.54%**	**10.00%**
Nominal Treasuries	**10%**	**54.17%**	**10.00%**
High Yield Bonds	**10%**	**0.00%**	**10.00%**
TIPS	**10%**	**0.00%**	**10.00%**
Commodities	**10%**	**0.00%**	**10.00%**

It is well-understood that asset class volatilities are not stationary, and dramatic changes in asset class correlations and spikes in asset class volatility are pervasive during market crises (see Appendix A, Exhibit A2). An illustration of correlation matrices generated using a 21-day half-life[10] exponentially weighted moving average as of November 20, 2008 can be seen in the bottom panel of Exhibit 2, alongside our familiar stable forecast estimated from 12 years of equally weighted data as of September 9,2009 for comparison. In the throes of the 2008 financial crisis, asset-class volatilities and correlations are substantially different than their long-term, equally weighted analogs. The use of a stressed covariance matrix to define scenarios and to run the constrained mean-variance optimization has a profound impact on the perturbed asset allocation.

Case 2: Allocating Assets in a Crisis Risk Climate

It is straightforward to create risk models that range in responsiveness by generating current covariance matrices using a variety of half-lives. However, if a manager believes that markets are on the precipice of a crisis, it may be prudent to go back in history to find a consistent covariance-matrix forecast. For example, a manager who believes that the US economy is on the brink of a prolonged period of disinflation might wish to impart this view to his asset allocation in the context of a stressed covariance matrix taken from a disinflationary historical period.

The financial crisis of 2008 spawned a disinflationary regime that led to deflation. To assess how asset class returns may behave during a period of disinflation or deflation, a manager can either construct a covariance matrix from an equally weighted sample of observations from relevant historical periods or take a historical, exponentially weighted moving average (EWMA) covariance matrix using an analysis date from the relevant regime. We take the latter approach

[10] A 21-day half life was chosen for this exercise, as this rate of decay has been empirically shown to provide accurate one-day variance forecasts.

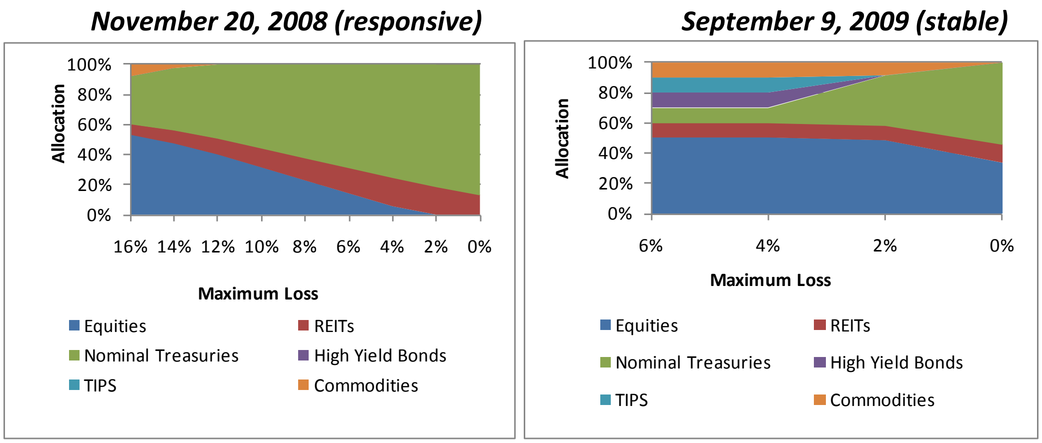
using the 21-day EWMA covariance matrix from November 20, 2008 shown in the bottom panel of Exhibit 2.

We re-examine the deflationary scenario defined in Case 1, but now analyze its impact using the responsive risk forecast described above. As expected, the scenario-implied profits and losses are much larger in magnitude in a crisis risk climate than in an average one. Notably, combining the explicit deflationary asset-class shock with a crisis risk climate leads to a sharp increase in allocation to government bonds—an unmistakable flight to quality. The crisis covariance matrix, with both its heightened asset class volatilities and extreme correlations, implies substantially larger losses and leads to a greater allocation to Nominal Treasuries, even at moderate loss levels. These results are summarized in Exhibits 5 and 6 below:

Exhibit 5: Implied alphas and Explicit and Implicit shocks for six US asset classes in a deflationary scenario. This table highlights the impact of the risk climate on implied alphas, and on implicit shocks generated by Formula (1). We apply a covariance matrix to the initial allocation in Exhibit 1 to generate implied alphas. Then, we apply the same covariance matrix to an explicit deflationary shock to Nominal Treasuries and TIPS, yielding implicit shocks to the other four asset classes. In the left panel, we use a responsive, 21-Day EWMA crisis covariance matrix as of November 20, 2008, which is shown in the bottom panel of Exhibit 2. In the right panel, we use the stable covariance as of September 9, 2009, which is shown in the top panel of Exhibit 2. The crisis covariance matrix, with both its heightened asset class volatilities and extreme correlations, implies substantially larger losses.

	Allocation implied alpha	November 20, 2008 (responsive)		Allocation implied alpha	September 9, 2009 (stable)	
		Explicit	Implicit		Explicit	Implicit
Equities	**0.18%**		**–27.36%**	**0.01%**		**–6.76%**
REITs	**0.26%**		**–32.31%**	**0.02%**		**–3.66%**
Nominal Treasuries	**–0.0098%**	**5.00%**	**5.00%**	**-.0007%**	**5.00%**	**5.00%**
High Yield Bonds	**0.015%**		**–12.50%**	**.0007%**		**–1.74%**
TIPS	**0.0006%**	**–5.00%**	**–5.00%**	**-.0005%**	**–5.00%**	**–5.00%**
Commodities	**0.016%**		**–8.87%**	**.0018%**		**-0.49%**

Exhibit 6: Perturbed Asset Allocations to six US Asset Classes in a deflationary scenario under varying constraints on maximum loss. This figure highlights the impact of the covariance matrix on the optimal allocation (determined by Formula (2) driven by data in Exhibit (5)). In the left panel, we use a responsive, 21-Day EWMA crisis covariance matrix as of November 20, 2008 (shown in the bottom panel of Exhibit 2). In the right panel, we use the stable covariance as of September 9, 2009, (shown in the top panel of Exhibit 2). The crisis covariance matrix, with both its heightened asset class volatilities and extreme correlations, leads to a greater allocation to Nominal Treasuries, even at moderate loss levels. (See Exhibit A4 in Appendix C for data underlying the charts.)



Case 3: Allocating Assets in a Hypothetical Risk Climate

We have seen that using historical risk climates may be more relevant to anticipated market behavior than a long-term average. We now show how to create hypothetical risk climates that fall outside the range of historical experience. Following Bender, Lee, and Stefek (2010), we can manipulate asset class correlations to reflect a manager's view. This is accomplished by introducing *a latent risk driver*, which selectively raises or lowers correlations between targeted asset classes without compromising the statistical integrity of the correlation matrix.[11]

For illustration, consider a stable forecast estimated from 10 years of equally weighted data as of November 20, 2008. Just at that time, Troubled Asset Relief Program (TARP)[12] was beginning to take effect, and some investors may have been concerned that inflation, not deflation, was on the horizon. Further, some may have believed that the (unstable) negative correlation between Equities and Nominal Treasuries that pervaded the early 2000s would not persist in the future, and assumed instead that the correlation between these two asset classes would revert to levels realized in the 1980s and mid-1990s of around 0.4.[13]

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[11]Modification of a correlation matrix can undermine its positive semi-definiteness, thereby making it unfit for statistical applications. In the framework outlined in Bender, Lee, and Stefek (2010), all asset class correlation adjustments are based on asset exposures to a latent variable. This procedure, maintains the positive semi-definiteness of the correlation matrix.

[12] TARP is a US Government program intended to strengthen the financial sector through the purchase of so-called troubled assets. The program was signed into law by President George W Bush on October 3, 2008.

[13] Bekaert et al. (2010) document the historical correlation between US Equities and Nominal Treasuries. Before 2000, the correlation was typically positive, relatively stable, and averaged around 0.4. After 2000, the correlation was less stable and often negative.

By giving both Equities and Nominal Treasuries positive exposure to a latent driver, we can modify the November 20, 2008 stable asset class correlation matrix to reflect the view that the correlation between these asset classes will return to the positive levels seen in a prior decade. Exhibit 7 shows volatilities and correlations before and after manipulation by assigning Equities and Nominal Treasuries an exposure of 0.75 to a latent driver[14]. The impact on the allocation decision can be seen by comparing the scenario-implied profits and losses in Exhibit 8, as well as allocations at various maximum loss levels in Exhibit 9. In the inflationary scenario based on the modified correlation matrix, the allocation to TIPS increases while the allocation to Equities decreases as the loss constraint tightens. As expected in an inflationary environment, Nominal Treasuries are eschewed.

Subtler effects, including an increased allocation to REITs, which showed a loss in this scenario, and a decreased allocation to Commodities, which are a standard inflation hedge, require deeper analysis. The explanation for these allocations lies in the implied alphas obtained through reverse optimization. Up to a constant of proportionality, implied alphas are marginal contributions of asset classes to portfolio volatility, and each marginal contribution is the product of the volatility of the asset class and its correlation with the portfolio[15]. Thus, the initial allocation affects the implied alphas. In our example, the REITs implied alpha is significantly larger than the Commodities implied alpha.

To illustrate the impact of the initial allocation, we retain the inflationary scenario described by the manipulated November 20, 2008 stable asset class correlation matrix, but now begin with an equally weighted allocation to the six asset classes. Exhibit 10 provides a comparison of implied alphas under the original and alternate initial allocations. In the alternate (equally weighted) allocation, the implied alpha for Equities is substantially lower, due to the reduction in correlation to the aggregate portfolio resulting from the reduced weight. A knock-on effect lowers the implied alpha for REITs, while the implied alpha for Commodities is higher. Exhibit 11 shows the dependence of perturbed allocations on the loss constraint using the equally weighted initial allocation. There is less tendency toward REITs and greater tendency toward Commodities than in the same scenario with the original initial allocation. Because of its profound impact on perturbed allocations, it is important to consider the initial allocation in the context of the scenario being defined.

[14] Bender, Lee and Stefek 's framework introduces a parameter ν_i to control the exposure of asset class i to a latent risk driver or unobservable source of commonality. They then use the following expression to manipulate the correlation between asset classes i and j,: $\rho_{i,j(new)} = \nu_i \nu_j + \sqrt{1-\nu_i^2}\sqrt{1-\nu_j^2}\rho_{i,j(original)}$

[15] Davis and Menchero (2010) illustrate that the marginal contribution to portfolio risk of a given source, traditionally defined as the partial derivative of portfolio volatility with respect to a change in source weight, can be intuitively expressed as the volatility of the source times the correlation of the source returns with the portfolio returns.

$MCR_i = \frac{\partial \sigma(R_P)}{\partial w_i} = \frac{\mathrm{cov}(r_i, R_P)}{\sigma(R_P)} = \sigma(r_i)\rho(r_i, R_P)$ Their risk attribution framework, built around this derivation, is aptly monikered x-sigma-rho.

Exhibit 7: Volatilities and correlation matrices for six US asset classes. This table highlights the latent variable technique for imposing views on an historical covariance matrix. The original covariance matrix in the top panel is as of November 20, 2008, and it is based on ten years of equally weighted daily data. The manipulated covariance matrix in the bottom panel is obtained by adding a latent driver that raises the correlation between Equities and Nominal Treasuries from the recent historical estimate of -0.27 to 0.44, which reflects empirically observed behavior in the 1990s.

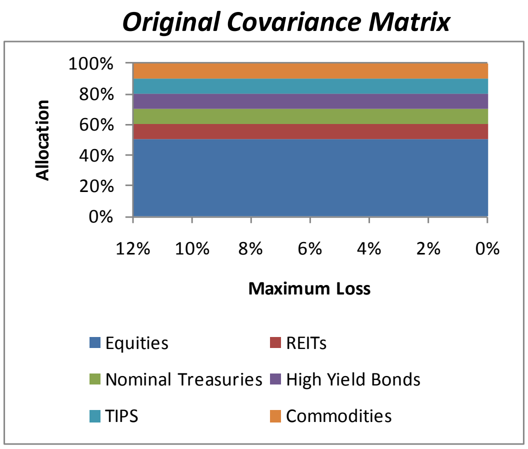
ORIGINAL	Volatility (annualized)	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Equities	**20.77%**	**1.00**					
REITs	**24.35%**	**0.61**	**1.00**				
Nominal Treasuries	**4.84%**	**-0.27**	**–0.17**	**1.00**			
High Yield Bonds	**4.74%**	**0.22**	**0.13**	**–0.03**	**1.00**		
TIPS	**5.44%**	**–0.18**	**–0.15**	**0.74**	**0.09**	**1.00**	
Commodities	**15.88%**	**0.00**	**0.02**	**-0.01**	**0.03**	**0.00**	**1.00**

MANIPULATED	Volatility (annualized)	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Equities	**20.77%**	**1.00**					
REITs	**24.35%**	**0.40**	**1.00**				
Nominal Treasuries	**4.84%**	**0.44**	**–0.11**	**1.00**			
High Yield Bonds	**4.74%**	**0.14**	**0.13**	**–0.02**	**1.00**		
TIPS	**5.44%**	**–0.12**	**–0.15**	**0.49**	**0.09**	**1.00**	
Commodities	**15.88%**	**0.00**	**0.02**	**-0.01**	**0.03**	**0.00**	**1.00**

Exhibit 8: Implied alphas and Explicit and Implicit shocks for six US asset classes in an inflationary scenario. This table highlights the impact of the risk climate on implied alphas, and on implicit shocks generated by Formula (1). We apply a covariance matrix to the initial allocation in Exhibit 1 to generate implied alphas. Then, we apply the same covariance matrix to an explicit deflationary shock to Nominal Treasuries and TIPS, yielding implicit shocks to the other four asset classes In the left panel, we use a stable, historical covariance matrix as of November 20, 2008, shown in the top panel of Exhibit 7. In the right panel, we use a manipulated version of this covariance matrix, shown in the bottom panel of Exhibit 7. The latent variable manipulation elevates the estimated correlation between Nominal Treasuries and TIPS to the level seen in the 1990s, changes the scenario Equity gain of 7.21% to a substantial loss.

	Allocation implied alpha	Original		Allocation implied alpha	Manipulated	
		Explicit	Implicit		Explicit	Implicit
Equities	**0.0145%**		**7.21%**	**0.0143%**		**-22.59%**
REITs	**0.0124%**		**1.68%**	**0.0094%**		**-1.64%**
Nominal Treasuries	**–0.0007%**	**-5.00%**	**-5.00%**	**.0015%**	**-5.00%**	**-5.00%**
High Yield Bonds	**0.007%**		**2.16%**	**.0005%**		**1.01%**
TIPS	**-0.0004%**	**5.00%**	**5.00%**	**-.0003%**	**5.00%**	**5.00%**
Commodities	**0.0015%**		**0.59%**	**.0015%**		**0.22%**

Exhibit 9: Perturbed Asset Allocations to six US Asset Classes in an inflationary scenario under varying under varying constraints on maximum loss. This figure highlights the impact of the risk climate on implied alphas, and on implicit shocks generated by Formula (1). In the left panel, we use a stable, historical covariance matrix as of November 20, 2008, shown in the top panel of Exhibit 7. In the right panel, we use a manipulated version of this covariance matrix, shown in the bottom panel of Exhibit 7. The latent variable manipulation elevates the estimated correlation between Nominal Treasuries and TIPS to the level seen in the 1990s. The inflationary scenario based on the original covariance does not produce a binding constraint and the initial allocation is unchanged. Under the inflationary scenario based on the manipulated covariance matrix, the allocation to TIPS increases while the allocation to Equities decreases as the loss constraint tightens. As expected in an inflationary environment, Nominal Treasuries are eschewed. (See Exhibits A5 and A6 in Appendix C for data underlying the charts.)



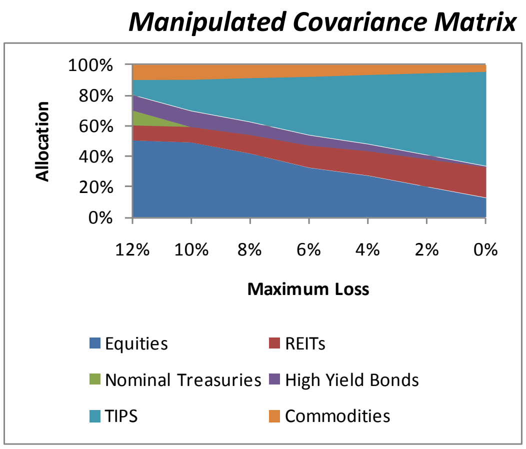
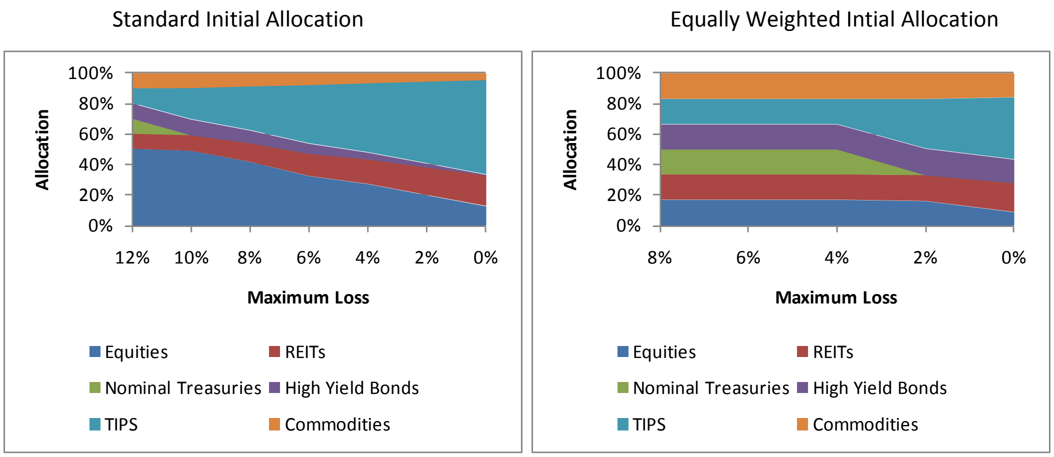


Exhibit 10: Implied alphas for six US asset classes in an inflationary scenario. This table highlights the impact of the initial allocation on implied alphas. We apply the manipulated stable covariance shown in the bottom panel of Exhibit 7. The first initial allocation is shown in the first column in the table below as well as in Exhibit 1, and it has been used in all previous examples. The second, equally weighted initial allocation is in the third column in the table below. In the equally weighted allocation, the implied alpha for Equities is substantially lower than in the original allocation, due to the reduction in correlation to the aggregate portfolio resulting from the reduced weight. A knock-on effect lowers the implied alpha for REITs, while the implied alpha for Commodities is higher.

	Initial Allocation	Allocation Implied Alpha	Initial Allocation	Allocation Implied Alpha
Equities	**50%**	**.0143%**	**16.67%**	**.0067%**
REITs	**10%**	**.0094%**	**16.67%**	**.0075%**
Nominal Treasuries	**10%**	**.0015%**	**16.67%**	**.0007%**
High Yield Bonds	**10%**	**.0005%**	**16.67%**	**.0003%**
TIPS	**10%**	**-.0003%**	**16.67%**	**.0001%**
Commodities	**10%**	**.0015%**	**16.67%**	**.0026%**

Exhibit 11: Perturbed Asset Allocations to six US Asset Classes in an inflationary scenario under varying constraints on maximum loss. This figure highlights the impact of the initial allocation on the optimal allocation (determined by Formula (2) driven by the covariance matrix in the bottom panel of Exhibit 7 and the implied alphas in Exhibit 10.) In the left panel, we use the initial allocation shown in Exhibit 1 and the first column in Exhibit 10. In the right panel, we use the equally-weighted initial allocation in the third column of Exhibit 10. There is less tendency toward REITs and Equities, and greater tendency toward Commodities than in the same scenario with the original initial allocation. (See Exhibits A6 and A7 in Appendix C for data underlying the charts.)



Case 4: Allocating Assets using Empirically Defined Shocks

In the previous examples, risk was determined by a covariance matrix, which was used to infer implicit asset class shocks from a specified explicit shock via the multivariate framework in Formula (1). Underlying this methodology is the assumption that asset class returns follow a Gaussian distribution. However, it is the tail scenarios, where traditional normality assumptions break down, that may be most relevant to stress testing. Accordingly, we broaden our scenario derivation process to take account of empirically observed extreme events.

An alternative to the Gaussian approach underlying Formula (1) is to define asset class profit and loss as the expected value of the return of that asset class, conditional on the explicit shocks exceeding a threshold. To forecast meaningfully the behavior of asset classes in the presence of shocks, we use a deep data history, and we apply the Barra Extreme Risk (BxR) methodology developed in Goldberg et al. (2008).[16] BxR recognizes that asset class volatility is non-stationary, so that the magnitude of an "extreme" return depends on the risk climate.

For example, a 3% loss in a single day may have been disastrous in May 2006, but it was relatively benign in October 2008. Therefore, we can create a more empirically grounded (and non-Gaussian) profit and loss scenario by scaling historical returns to a desired covariance climate. The expected value of each normalized asset class return, conditional on the shocked variables exceeding a specified threshold, is estimated directly from the rescaled data series, without making parametric assumptions. It is important to highlight that while our empirical approach makes no parametric assumption about

[16] As documented in Dubikovsky et al. (2010), this methodology leads to accurate forecasts of shortfall for a wide range of international equity portfolios.

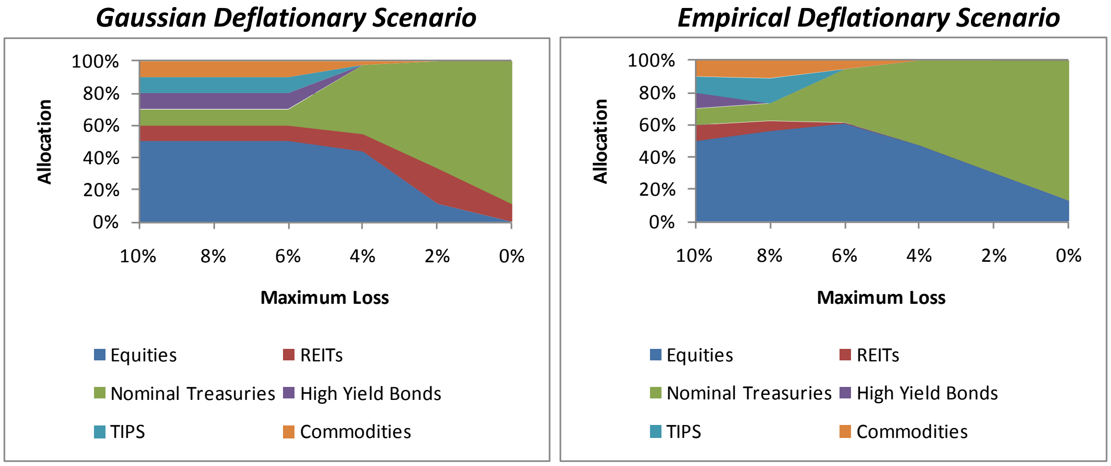
the expected P&L for a given scenario, the estimate is based upon re-scaled historical returns that are directly impacted by a covariance-defined risk climate. Importantly, the magnitude of rescaled returns, and hence of asset class expected profit and loss, depends critically upon the covariance climate chosen. Exhibit 12 displays the asset class profits and losses for a scenario whereby TIPS fall by 3%, and Nominal Treasuries simultaneously rise by 1%.

First, we derive the returns in the Gaussian framework from Formula (1) using a 21-day EWMA covariance matrix as of November 20, 2008. Second, we use the empirically defined approach described above, using the November 20, 2008 21-day EWMA covariance matrix as the climate for re-scaling. Specifically, we estimate the expected value as of November 20, 2008 of each asset class, conditional on both (covariance-scaled) Nominal Treasuries rising by at least 1% and (covariance-scaled) TIPS falling by 3%, over the entire history of observations. The profits and losses vary substantially between these two approaches. In particular, the empirical approach, which is more sensitive to extreme events, returns a substantially larger loss for REITs and High Yield Bonds than does the Gaussian distribution. This leads to a dramatic difference in perturbed allocations as shown in Exhibit 13.

Exhibit 12: Explicit and Implicit shocks for six US asset classes in a deflationary scenario. This table highlights the impact of the Gaussian assumption underlying Formula (1). We explicitly apply a deflationary shock of a 1% increase to Nominal Treasuries and a 3% decrease to TIPS in the crisis risk climate determined by the 21-day halflife EWMA covariance matrix as of November 20, 2008, shown in the bottom panel of Exhibit 2. The left panel shows Gaussian Implicit shocks generated by Formula (1), and the right panel shows Empirical implicit shocks. The empirical approach, which is more sensitive to extreme events, returns a substantially larger loss for REITs and High Yield Bonds than does the Gaussian distribution.

	Gaussian		Empirical	
	Explicit	Implicit	Explicit	Implicit
Equities		**–8.00%**		**–10.13%**
REITs		**–7.95%**		**–22.44%**
Nominal Treasuries	**1.00%**	**1.00%**	**1.00%**	**1.48%**
High Yield Bonds		**–4.69%**		**–12.48%**
TIPS	**–3.00%**	**–3.00%**	**–3.00%**	**–3.91%**
Commodities		**–2.57%**		**–3.85%**

Exhibit 13: Perturbed Asset Allocations to six US Asset Classes in a deflationary scenario under varying constraints on maximum loss. This figure highlights the impact of the Gaussian assumption on the optimal allocation (determined by Formula (2) driven by the covariance matrix in the bottom panel of Exhibit 2 and the implied alphas in the first column of Exhibit 5.) In the left panel, we use the Gaussian scenario in the left panel of Exhibit 12. In the right panel, we use Empirical scenario in the right panel of Exhibit 12. The Empricial shocks diminish allocations to risky asset classes under milder constraints. (See Exhibits A8 and A9 in Appendix C for data underlying the charts.)



Case 5: Comparing Asset Allocations using Gaussian and Empirically Defined Shocks under Combat Conditions

In our final example, we explore the impact that extreme risk climate asset allocation might have had in the darkest moments of the financial crisis. We emphasize the differences that stem from using empirical- versus Gaussian-implied shocks. Note that the examples presented below serve only as illustrations and do not constitute a thorough statistical evaluation.

By June 2008, market participants had weathered a year of high volatility and unprecedented extreme events. Some may have sought a relatively conservative investment strategy that would perform well in a flight-to-quality. Consider a hypothetical flight-to-quality risk climate with volatility, say, at three times the level observed at the end of June 2008. Further, we assume relatively high correlation across the five riskier asset classes, and will specify this assumption by supposing all are pairwise correlated at .7. Finally, we assume that the traditional safe haven, Nominal Treasuries, moves in the opposite direction from riskier asset classes with a correlation coefficient of -0.3.[17] This hypothetical risk climate, meant only to be a naïve schematic, is summarized in Exhibit 14.

We retain the deflationary shock used in Case 4, a 1% increase in Nominal Treasuries and a 3% decline in TIPS, and show in Exhibit 15, the implied asset class profits and losses under the Gaussian and empirical models using the hypothetical risk climate. Note that while they are a function of the same risk climate, the implied loss to Equities is much greater under the empirical model than under the Gaussian model, empirically suggesting that Equities may exhibit tails that are heavier than that which is implied by the Gaussian distribution.

[17] This hypothetical regime is inspired by the events of October of 2008.

The corresponding perturbed asset allocations are shown in Exhibit 16. The shocks implied by the empirical approach lead to modifications of the initial asset allocation under weaker loss constraints. Under tighter constraints, the Gaussian model allocates almost completely to Nominal Treasuries, while the empirical model allocation is largely a split between Nominal Treasuries and Commodities. In Exhibit 17, we examine the performance of the asset allocations over the turbulent six-month period from June to December 2008. Since the 6% loss constraint produced by the Gaussian model was not binding, it left the initial allocation untouched and resulted in a return of -25%. The empirical model generated a binding constraint that moved assets from Equities, REITS and High Yield Bonds to TIPS, Commodities and Nominal Treasuries. The result was a better performance at -19%. The 4% loss constraint was binding for both models, and resulted in returns of -25% for the Gaussian model and -17% for the empirical model. When losses were more tightly constrained to 2% or 0%, the empirical model underperformed the Gaussian, which concentrated assets in the only class that performed well during the period from June through December 2008.

It turns out that investors who were able to accurately envision the November 20, 2008 risk climate in June 2008 would have been in a fortuitous position. Even without precise knowledge of the future asset class returns, those who could accurately forecast the covariance regime prevailing through the fall of 2008, would have been poised to benefit substantially. We examined the performance of the perturbed allocations displayed in Exhibit 13 for the six-month period extending from June through December 2008. These allocations were generated by using the Gaussian and empirical models calibrated to the November 20, 2008 crisis covariance matrix to generate shocks to all asset classes based on a 1% increase in Nominal Treasuries and a 3% decrease in TIPS. As shown in Exhibit 18, the empirical model outperformed the Gaussian model at all levels of loss constraint, and both models outperformed the initial allocation whenever the constraint was binding.

Exhibit 14: Hypothetical Covariance Matrix. Volatilities and correlations for six US asset classes in a flight-to-quality in which volatilities are relatively high, risky assets are relatively correlated with one another and anti-correlated with Nominal Treasuries.

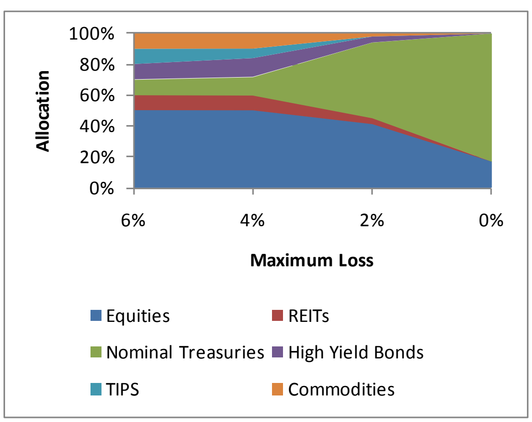
HYPOTHETICAL	Volatility (annualized)	Equities	REITs	Nominal Treasuries	High Yield Bonds	TIPS	Commodities
Equities	**56.04%**	**1.00**					
REITs	**92.16%**	**0.7**	**1.00**				
Nominal Treasuries	**17.33%**	**-0.3**	**-0.3**	**1.00**			
High Yield Bonds	**12.11%**	**0.7**	**0.7**	**–0.3**	**1.00**		
TIPS	**24.59%**	**0.7**	**0.7**	**-0.3**	**0.7**	**1.00**	
Commodities	**64.37%**	**0.7**	**0.7**	**-0.3**	**0.7**	**0.7**	**1.00**

Exhibit 15: Implied Alphas and Explicit and Implicit shocks for six US asset classes in a deflationary scenario. This table highlights the impact of the Gaussian assumption underlying Formula (1). We explicitly apply deflationary shock of a 1% increase to Nominal Treasuries and a 3% decrease to TIPS in the hypothetical risk climate shown in Exhibit 14. The left panel shows Gaussian Implicit shocks generated by Formula (1), and the right panel shows Empirical implicit shocks. The implied alphas are reverse optimized from the hypothetical covariance matrix in Exhibit 14 and the initial allocation in Exhibit 1. The empirical approach, which is more sensitive to extreme events, returns a substantially larger loss for Equities than does the Gaussian distribution.

	Allocation implied alpha	Gaussian		Empirical	
		Explicit	Implicit	Explicit	Implicit
Equities	**.1719%**		**–4.90%**		**–15.41%**
REITs	**.2110%**		**–8.06%**		**–8.84%**
Nominal Treasuries	**-.0119%**	**1.00%**	**1.00%**	**1.00%**	**2.03%**
High Yield Bonds	**.0312%**		**–1.06%**		**–3.04%**
TIPS	**.0351%**	**–3.00%**	**–3.00%**	**–3.00%**	**–3.47%**
Commodities	**.1085%**		**–5.63%**		**–2.88%**

Exhibit 16: Perturbed Asset Allocations to six US Asset Classes in a deflationary scenario under varying constraints on maximum loss. This figure highlights the impact of the Gaussian assumption on the optimal allocation (determined by Formula (2) driven by the covariance matrix in Exhibit 14 and the implied alphas in the first column of Exhibit 15.) In the left panel, we use the Gaussian scenario in the left panel of Exhibit 15. In the right panel, we use Empirical scenario in the right panel of Exhibit 15. Under tighter constraints, the Gaussian model allocates almost completely to Nominal Treasuries, while the empirical model allocation is largely a split between Nominal Treasuries and Commodities. See Exhibits A10 and A11 in Appendix C for data underlying the charts.)

Gaussian Deflationary Scenario



Empirical Deflationary Scenario

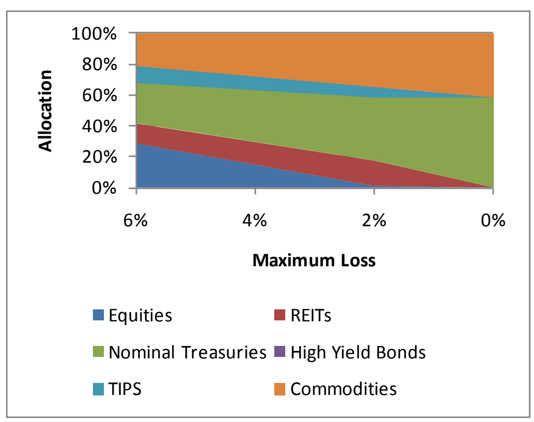


Exhibit 17: Six-month Performance (June 30, 2008-December 31, 2008) of perturbed allocations at various levels of maximum loss. This figure highlights the impact of the Gaussian assumption on performance during the darkest moments of the financial crisis. We show the realized performance of the Gaussian and Empirical allocations shown in Exhibit 16. Empirical allocations outperform at weaker constraints, while the Gaussian allocations outperform at tighter constraints.

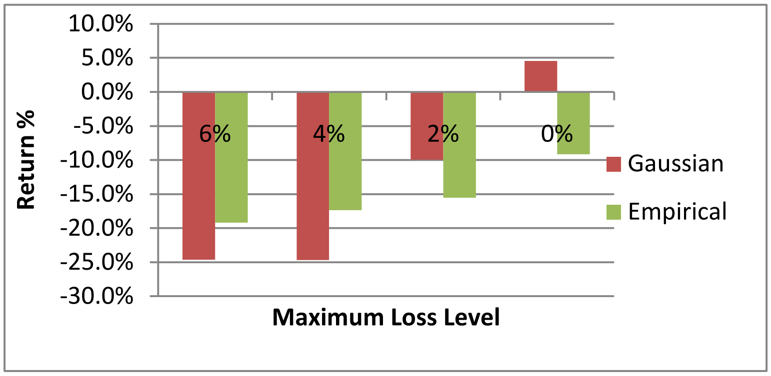
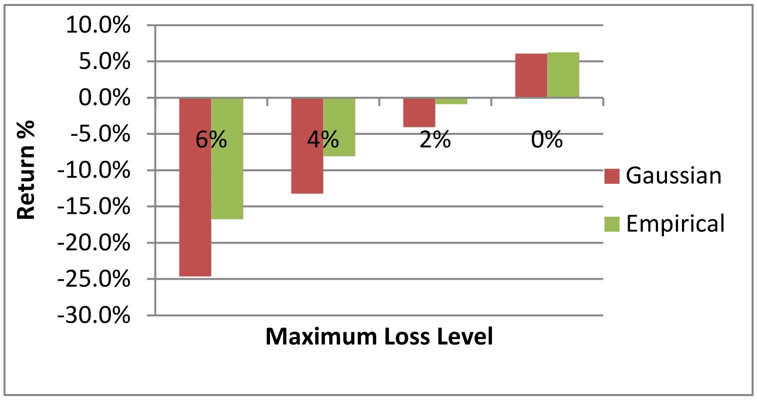


Exhibit 18: Six-month Performance (June 30, 2008-December 31, 2008) of perturbed allocations at various levels of maximum loss. This figure highlights the value of perfect foresight into future risk climates. We use the initial allocation in Exhibit 1, the crisis covariance matrix as of November 20, 2008 in the bottom panel of Exhibit 2 and both Gaussian and Empirical assumptions to imply deflationary scenarios corresponding to a 1% gain to Nominal treasuries and a 3% loss to TIPS. Empirical allocations outperform at all constraints examined.



Conclusion

While it may not be possible to precisely measure the likelihood of an extreme event, it is nevertheless a valuable exercise to assess its impact on a portfolio. In this article, we introduced a new paradigm for translating extreme events into asset-class scenarios. An essential point is that the risk climate plays an integral role in the translation. Even within a Gaussian framework, a wide range of risk climates can be obtained from history by varying the analysis date and the responsiveness of covariance matrix estimation. Furthermore, any historical covariance matrix can be modified to reflect exogenous views on future correlations via the introduction of a latent factor, which can represent a flight to quality, a change in liquidity, or another transient effect that disrupts markets in a crisis. Importantly, the latent factor preserves positive semi-definiteness, and hence the essential statistical character of the covariance matrix. More broadly, risk climates can be empirically specified in terms of expected values of asset classes in an extreme situation. Anecdotally, implicit shocks generated by empirical distributions tend to be larger than the Gaussian analogs. A more systematic analysis is a topic for future research.

A second contribution made in this Research Insight is a quantitative method to modify asset allocation weights in a stress scenario. A scenario-constrained optimization using reverse-optimized alphas modifies an initial asset allocation in intuitive and conservative ways. It is important to highlight the flexibility of the approach outlined here. The quadratic formulation outlined in Formula (2) can easily be modified to include additional constraints (such as bounds on weights or portfolio turnover) in order to make the results more applicable to an individual's existing allocation process. It can also be expanded to include a constraint or penalty for extreme risk, as discussed in Bender, et al (2010) and Goldberg, et al (2011).

Whether used in combination or alone, these contributions are material extensions of the standard stress testing methodology and they can provide investors of all types with valuable input to their decisions.

Acknowledgements

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Appendix A: Historical Asset Class Correlations and Volatilities

Exhibit A1: Correlation between Treasuries and TIPS, 21-Day EWMA

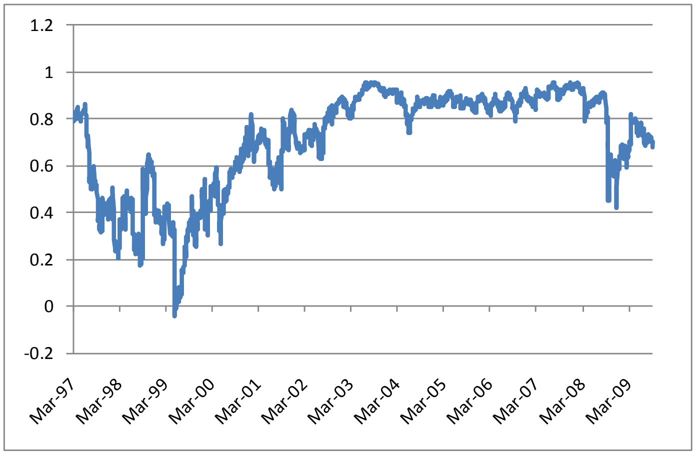
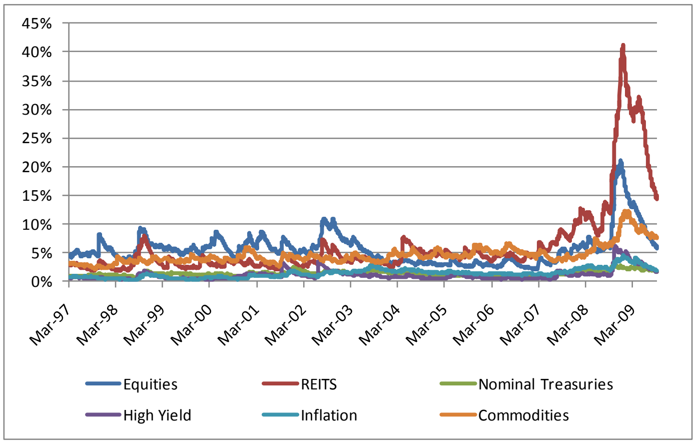


Exhibit A2: Asset Class Volatility, 21-Day EWMA



Appendix B: Data Proxies

Exhibit A3: Representative Index Portfolio per Asset Class

Asset class	Representative Index Portfolio
Equities	**MSCI USA**
REITs	**MSCI US REIT**
Nominal Treasuries	**Merrill Lynch US Treasury**
High Yield Bonds	**Merrill Lynch US High Yield**
TIPS	**Merrill Lynch US Inflation Linked**
Commodities	**Dow Jones UBS Commodity**

Appendix C: Modified Asset Class Weights Under Stress Scenarios

Exhibit A4: Perturbed Allocations, 21-Day EWMA and Equal Weighted stable Covariance forecast

	Maximum loss from deflationary scenario (Responsive, November 20, 2008)									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**53.3%**	**47.5%**	**40.1%**	**31.5%**	**22.8%**	**14.2%**	**5.6%**	**0.0%**	**0.0%**
REITs	**10.0%**	**7.2%**	**8.9%**	**10.8%**	**12.9%**	**15.0%**	**17.2%**	**19.3%**	**18.8%**	**13.4%**
Nominal Treasuries	**10.0%**	**31.7%**	**41.3%**	**49.1%**	**55.6%**	**62.1%**	**68.6%**	**75.1%**	**81.2%**	**86.6%**
High Yield Bonds	**10.0%**	**0.0%**	**0.0%**	**0.0%**	**0. 0%**	**0.0%**	**0.0%**	**0. 0%**	**0. 0%**	**0. 0%**
TIPS	**10.0%**	**0.0%**	**0.0%**	**0.0%**	**0. 0%**	**0.0%**	**0.0%**	**0. 0%**	**0. 0%**	**0. 0%**
Commodities	**10.0%**	**7.9%**	**2.4%**	**0.0%**	**0. 0%**	**0.0%**	**0.0%**	**0.0%**	**0. 0%**	**0. 0%**
	Maximum loss from deflationary scenario (Stable September 9, 2009)									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50. 0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**48.2%**	**33.3%**
REITs	**10. 0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.1%**	**12.5%**
Nominal Treasuries	**10. 0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**33.3%**	**54.2%**
High Yield Bonds	**10. 0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0. 0%**	**0.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0.0%**	**0.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**8.5%**	**0.0%**

Exhibit A5: Perturbations using Original Covariance Matrix, Stable (November 20, 2008)

	Maximum loss from inflationary scenario									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**

Exhibit A6: Perturbations using Manipulated Covariance Matrix, Stable (November 20, 2008)

	Maximum loss from inflationary scenario									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**48.7%**	**41.4%**	**31.1%**	**26.9%**	**19.6%**	**12.4%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.4%**	**12.4%**	**14.3%**	**16.3%**	**18.3%**	**20.2%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.4%**	**8.4%**	**6.5%**	**4.5%**	**2.6%**	**0.6%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**20.7%**	**29.0%**	**37.3%**	**45.6%**	**53.9%**	**62.2%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**9.9%**	**8.8%**	**7.7%**	**6.7%**	**5.6%**	**4.6%**

Exhibit A7: Perturbations using Manipulated Covariance Matrix, Stable (November 20, 2008) and equally weighted initial allocation

	Maximum loss from inflationary scenario									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**15.8%**	**8.6%**
REITs	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**17.0%**	**19.0%**
Nominal Treasuries	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**0.0%**	**0.0%**
High Yield Bonds	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**17.7%**	**15.7%**
TIPS	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**32.8%**	**41.2%**
Commodities	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**16.7%**	**15.6%**

Exhibit A8: Perturbed Allocations, 21-Day EWMA Covariance Forecast (November 20, 2008) Gaussian-Implied Deflationary Scenario

	Maximum loss from deflationary scenario, covariance based									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**43.5%**	**11.2%**	**0.0%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**11.2%**	**22.2%**	**11.2%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**43.0%**	**66.5%**	**88.8%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10. 0%**	**10.0%**	**0.0%**	**0.0%**	**0.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0.0%**	**0.0%**	**0.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**2.3%**	**0.0%**	**0.0%**

Exhibit A9: Perturbed Allocations, 21-Day EWMA Covariance Forecast (November 20, 2008) Empirically Defined Deflationary Scenario

	Maximum loss from deflationary scenario, empirical									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**56.3%**	**61.0%**	**47.2%**	**30.0%**	**12.8%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**6.3%**	**0.5%**	**0.0%**	**0.0%**	**0.0%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.7%**	**33.1%**	**52.8%**	**70.0%**	**87.2%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0.00%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**15.6%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**11.1%**	**5.4%**	**0.0%**	**0.0%**	**0.0%**

Exhibit A10: Perturbed Allocations, Hypothetical Risk Climate, Gaussian Implied Deflationary Scenario

	Maximum loss from deflationary scenario, Gaussian									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**49.9%**	**41.0%**	**16.9%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**9.9%**	**4.4%**	**0.0%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**11.9%**	**49.0%**	**83.1%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10. 0%**	**10.0%**	**12.2%**	**3.9%**	**0.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**6.2%**	**0.0%**	**0.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**9.9%**	**2.0%**	**0.0%**

Exhibit A11: Perturbed Allocations, Hypothetical Risk Climate, Empirically Defined Deflationary Scenario

	Maximum loss from deflationary scenario, empirical									
	Starting allocation	16%	14%	12%	10%	8%	6%	4%	2%	0%
Equities	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**50.0%**	**42.3%**	**28.6%**	**14.8%**	**1.0%**	**0.0%**
REITs	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**11.1%**	**12.9%**	**14.6%**	**16.4%**	**0.0%**
Nominal Treasuries	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**18.6%**	**26.3%**	**33.8%**	**41.2%**	**58.6%**
High Yield Bonds	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**0.7%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
TIPS	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**13.0%**	**11.1%**	**8.9%**	**6.8%**	**0.0%**
Commodities	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**10.0%**	**14.4%**	**21.1%**	**27.9%**	**34.7%**	**41.4%**

Appendix D: A Chronicle of Recent Breakeven Inflation in the United States

Exhibits A12 and A13 show US nominal and real interest rate term structures on a series of dates at six-month intervals. The earliest curve shown, July 7, 2007, predates the recent crisis and indicates flat nominal and real rates at roughly 5% and 3%, respectively. Later in July, however, troubles at two Bear Stearns hedge funds specializing in subprime mortgages became public, setting off market disruptions that foreshadowed the crisis that followed. The market responded with a flight to quality. Rates dropped by roughly 2% at the short end and 0.5% at the long end, leaving an upwardly sloping yield curve as of January 4, 2008.

In the first half of 2008, increased commodity and oil prices led to a bout of inflation that troubled consumers in the form of increasing prices for gasoline, shipping, and air travel. In March, Bear Stearns was sold to JP Morgan at a fire-sale price. Markets were volatile, nominal interest rates held steady, and real interest rates dropped at the short end. Autumn 2008 witnessed the unprecedented deflationary shock featured in this article. The Lehman Brothers collapse triggered a dramatic flight to nominal US Treasuries, and TIPS were eschewed along with equities, REITS, commodities, and other risky securities. As the US rolled out its Troubled Assets Relief Program (TARP), investors began to worry about potential inflation and reversed their positions on TIPS, and there was a return to normalcy in the first half of 2009. US breakeven inflation rates corresponding to the four 6-month intervals between July 2007 and July 2009 are shown in Exhibits A14.

Exhibit A12: US Nominal Rates

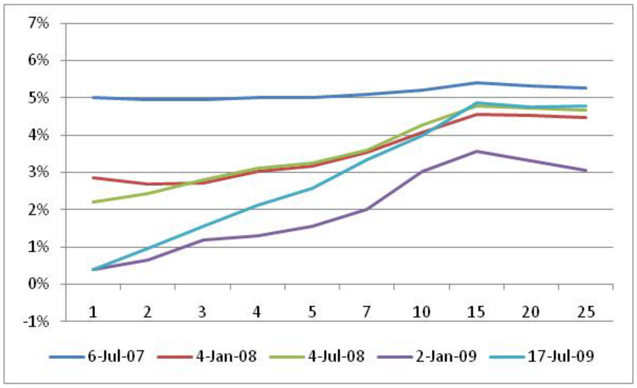


Exhibit A13: US Real Rates

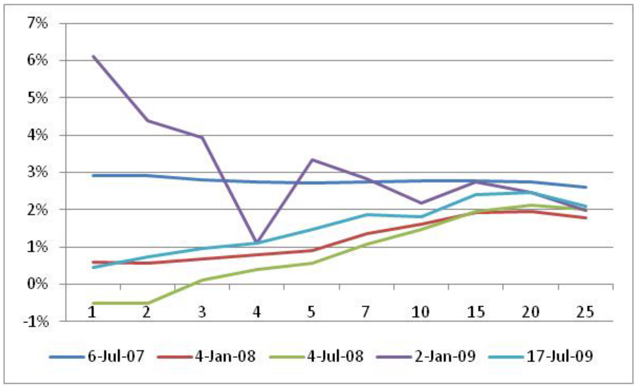
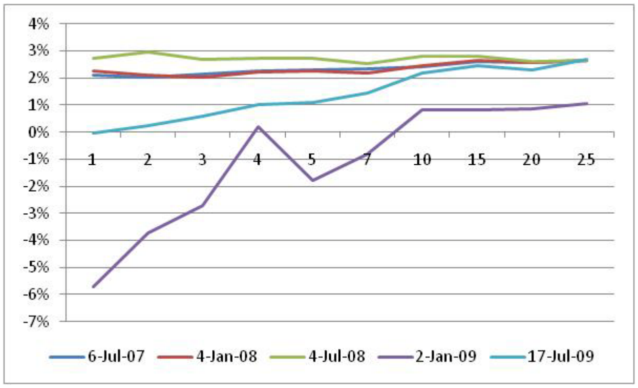


Exhibit A14: US Breakeven Inflation Rates



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